

COMPACT WEIGHTED COMPOSITION OPERATORS AND FIXED POINTS IN CONVEX DOMAINS

DANA D. CLAHANE

ABSTRACT. We extend a classical result of Caughran/H. Schwartz and another recent result of Gunatillake by showing that if D is a bounded, convex domain in \mathbb{C}^n , $\psi : D \rightarrow \mathbb{C}$ is analytic and bounded away from zero toward the boundary of D , and $\phi : D \rightarrow D$ is a holomorphic map such that the weighted composition operator $W_{\psi,\phi}$ is compact on a holomorphic, separable, functional Hilbert space (containing the polynomial functions densely) on D with reproducing kernel K satisfying $K(z, z) \rightarrow \infty$ as $z \rightarrow \partial D$, then ϕ has a unique fixed point in D . We apply this result by making a reasonable conjecture about the spectrum of $W_{\psi,\phi}$ based on previous results.

1. INTRODUCTION

Let ϕ be a holomorphic self-map of a bounded domain D in \mathbb{C}^n , and suppose that ψ is a holomorphic function on D . We define the linear operator $W_{\psi,\phi}$ on the linear space of complex-valued, holomorphic functions $\mathcal{H}(D)$ by

$$W_{\psi,\phi}(f) = \psi(f \circ \phi).$$

$W_{\psi,\phi}$ is called the *weighted composition operator* induced by the *weight symbol* ψ and *composition symbol* ϕ . Note that $W_{\psi,\phi}$ is the (unweighted) composition operator C_ϕ given by $C_\phi(f) = f \circ \phi$ when $\psi = 1$.

Weighted composition operators are not only fundamental objects of study in analysis, but they have recently been the subject of an increasing amount of attention due to the fact that a rather vast amount of results have been obtained about unweighted composition operators in one and several complex variables. It should also be pointed out that by the Banach-Stone Theorem, the isometries between the spaces of continuous functions on Hausdorff spaces turn out to be weighted

Date: August 25, 2005.

2000 Mathematics Subject Classification. Primary: 47B38; Secondary: 26A16, 32A16, 32A26, 32A30, 32A37, 32A38, 32H02, 47B33.

Key words and phrases. weighted composition operator, bounded operator, compact operator, convex domain, Hilbert space, weighted Hardy space, Bergman space, holomorphic, several complex variables.

composition operators [S]. This phenomenon also occurs when these spaces are replaced by other function spaces on domains in \mathbb{C} or \mathbb{C}^n (cf. [B], [CiW], [dRW], [FIJ], [Fo]) and continues to be studied.

It is natural to consider the dynamics of the sequence of iterates of a composition symbol of a weighted composition operator and the spectra of such operators. The following classical result of Caughran/Schwartz [CaSc] began this line of investigation.

Theorem 1.1. *Let $\phi : \Delta \rightarrow \Delta$ be an analytic self-map of the unit disk Δ in \mathbb{C} . If C_ϕ is compact or power-compact on the Hardy space $H^2(\Delta)$, then the following statements hold:*

- (a) ϕ must have a unique fixed point in Δ (this point turns out to be the so-called Denjoy-Wolff point a of ϕ in Δ (see [CowMac, Ch. 2]).
- (b) The spectrum of C_ϕ is the set consisting of 0, 1, and all powers of $\phi'(a)$.

The analogue of this result for Hardy spaces of the unit ball \mathbb{B}_n in \mathbb{C}^n was obtained by MacCluer in [Mac2]:

Theorem 1.2. *Let $\phi : \mathbb{B}_n \rightarrow \mathbb{B}_n$ be a holomorphic self-map of \mathbb{B}_n , and suppose that $p \geq 1$. If C_ϕ is compact or power compact on the Hardy space $H^p(\mathbb{B}_n)$, then*

- (a) ϕ must have a unique fixed point in \mathbb{B}_n (again, this point is the so-called Denjoy-Wolff point a of ϕ in \mathbb{B}_n (see [CowMac, Ch. 2]).
- (b) The spectrum of C_ϕ is the set consisting of 0, 1, and all products of eigenvalues of $\phi'(a)$.

This result also holds for weighted Bergman spaces of \mathbb{B}_n [CowMac]. The proofs of parts (a) of Theorems 1.1 and 1.2 appeal to the Denjoy-Wolff theorems in Δ and \mathbb{B}_n . It is natural to consider whether Theorem 1.1 holds when \mathbb{B}_n is replaced by more general bounded symmetric domains or even the polydisk Δ^n . Chu/Mellon in [ChMe] recently showed that the Denjoy-Wolff theorem fails in Δ^n for $n > 1$; nevertheless, it is shown in [Cl2] that MacCluer's results can be generalized from \mathbb{B}_n to arbitrary bounded symmetric domains that are either reducible or irreducible.

G. Gunatillake in the forthcoming paper [G] has extended Theorem 1.1 to weighted composition operators on a certain class of weighted Hardy spaces of Δ when ψ is bounded away from 0 toward the unit circle in \mathbb{C} :

Theorem 1.3. *Let $(b_j)_{j \in \mathbb{N}}$ be a sequence of positive numbers such that $\liminf_{j \rightarrow \infty} b_j^{1/j} \geq 1$, and let $H_b^2(\Delta)$ be the weighted Hardy space of analytic functions $f : \Delta \rightarrow \mathbb{C}$ whose MacClaurin series $f(z) = \sum_{j=0}^{\infty} a_j z^j$*

satisfy $\sum_{j=0}^{\infty} |a_j|^2 b_j^2 < \infty$. Suppose that $\phi : \Delta \rightarrow \Delta$ is analytic, and let $\psi : \Delta \rightarrow \mathbb{C}$ be an analytic map that is bounded away from zero toward the unit circle. Assume that $W_{\psi,\phi}$ is compact on $H_b^2(\Delta)$. Then the following statements are true:

- (a) ϕ has a unique fixed point $a \in \Delta$.
- (b) The spectrum of $W_{\psi,\phi}$ is the set

$$\{0, \psi(a)\} \cup \{\psi(a)[\phi'(a)]^j : j \in \mathbb{N}\}.$$

Related results for unweighted Hardy spaces and an example $W_{\psi,\phi}$ that is compact even though C_ϕ is not appear in [ShSm].

In what follows, we will introduce some basic notation. Then, we will make a conjecture concerning a multivariable analogue of the above result. The purpose of this paper is to report that part (a) of Theorem 1.3 extends quite generally to a wide class of functional Hilbert spaces on convex domains in one or more variables. The calculation of the spectrum of a compact $W_{\psi,\phi}$ in the multivariable setting is the subject of future work.

Some of the following notation and definitions are standard, but we include them for the sake of clarity and completeness:

2. NOTATION AND DEFINITIONS

Fix $n \in \mathbb{N}$. We denote the usual Euclidean distance from $z \in \mathbb{C}^n$ to $A \subset \mathbb{C}^n$ by $d(z, A)$, and we say that $z \rightarrow A$ if and only if $d(z, A) \rightarrow 0$. Let D be a bounded domain in \mathbb{C}^n . Suppose that $\nu \in \mathbb{R}$, and assume that $u : D \rightarrow \mathbb{R}$. We write

$$\liminf_{z \rightarrow \partial D} u(z) = \nu$$

if and only if for all $\varepsilon > 0$ there is a $\delta > 0$ such that whenever $0 < d(z, \partial D) < \delta$, we have

$$\left| \inf_{\{z \in D : d(z, \partial D) < \delta\}} u(z) - \nu \right| < \varepsilon.$$

Let $\psi : D \rightarrow \mathbb{C}$. We say that ψ is *bounded away from zero toward the boundary of D* iff there is a $\nu > 0$ such that

$$\liminf_{z \rightarrow \partial D} |\psi(z)| = \nu.$$

Given a functional Hilbert space \mathcal{Y} of holomorphic functions defined on a domain D , in \mathbb{C}^n , the Riesz representation theorem guarantees that for each $z \in D$, there is a unique $K_z \in \mathcal{Y}$ such that

$$f(z) = \langle f, K_z \rangle \quad \text{for all } f \in \mathcal{Y}.$$

This uniqueness ensures allows us to define the *reproducing kernel* $K : D \times D \rightarrow \mathbb{C}$ for \mathcal{Y} by $K(z, w) = K_z(w)$.

3. A CONJECTURE AND THE RESULT

Based on the results to date, we propose the natural problem of resolving whether or not the following statement holds:

Conjecture: *Suppose that $D \subset \mathbb{C}^n$ is a bounded, convex domain such that a given functional Hilbert space of holomorphic functions \mathcal{Y} in which the polynomials are contained densely has reproducing kernel K satisfying $K(z, z) \rightarrow \infty$ as $z \rightarrow \partial D$. Let $\psi : D \rightarrow \mathbb{C}$ be holomorphic and suppose that ψ is bounded away from 0 toward ∂D . Assume that $\phi : D \rightarrow D$ is a holomorphic map and that $W_{\psi, \phi}$ is compact on \mathcal{Y} . Then*

- (1) ϕ has a unique fixed point in D , and
- (2) the spectrum of $W_{\psi, \phi}$ is the set $\{\psi(a)\sigma : \sigma \in E\}$, where E is the set consisting of 0, 1, and all possible products of eigenvalues of $\phi'(a)$.

While (2) has not yet been resolved in the multivariable case, we are able to report that item (1) indeed holds. What is unique about this result is that most results about properties of composition operators have heavily depended on the function space under consideration; contrastingly, by proving (1), we obtain a result that is relatively independent of the given Hilbert space.

The following theorem generalizes results in [CaSc], [G], and, in one direction, the fixed point portion of [Cl2, Thm. 4.2]. The reader is referred to [Cl2] for the definition of compact divergence, which is used in the proof that follows.

Theorem 3.1. *Let $D \subset \mathbb{C}^n$ be a convex domain, and suppose that \mathcal{Y} is a functional Hilbert space of functions on D with reproducing kernel $K : D \times D \rightarrow \mathbb{C}$. Assume that $K(z, z) \rightarrow \infty$ as $z \rightarrow \partial D$, and assume that the polynomial functions on D are dense in \mathcal{Y} . Suppose that $\psi : D \rightarrow \mathbb{C}$ is holomorphic and bounded away from zero toward the boundary of D , and let $\phi : D \rightarrow D$ be holomorphic. Assume that $W_{\psi, \phi}$ is compact on \mathcal{Y} . Then ϕ has a unique fixed point in D .*

Proof. Let $k_z = K_z / \|K_z\|_{\mathcal{Y}}$. Since $K(z, z) \rightarrow \infty$ as $z \rightarrow \partial D$ and the polynomial functions on D are dense in \mathcal{Y} , one can show using an argument identical to that of the proof of [Cl2, Lemma 3.1] that $k_z \rightarrow 0$ weakly as $z \rightarrow \partial D$. From the linearity of $W_{\psi, \phi}$ and the identity

$$W_{\psi, \phi}^* K_z = \overline{\psi(z)} K_{\phi(z)},$$

it immediately follows that

$$\|W_{\psi,\phi}^* k_z\|_{\mathcal{Y}}^2 = |\psi(z)|^2 K(z, z)^{-1} K[\phi(z), \phi(z)].$$

Since $k_z \rightarrow 0$ weakly as $z \rightarrow \partial D$, we then have that

$$(1) \quad \lim_{z \rightarrow \partial D} |\psi(z)|^2 K(z, z)^{-1} K[\phi(z), \phi(z)] = 0.$$

First suppose that ϕ has no fixed point in D . We will obtain a contradiction. Let $z \in D$. Since D is convex, the sequence of iterates $\phi^{(j)}$ of ϕ is compactly divergent [Ab, p. 274]. Thus, for every compact $K \subset D$ there is an $N \in \mathbb{N}$ such that $\phi^{(j)}(z) \in D \setminus K$ for all $j \geq N$.

Since for any $\varepsilon > 0$, the set K_ε of all $w \in D$ such that $d(w, \partial D) \geq \varepsilon$ is compact, it follows from the statement above that for all $\varepsilon > 0$ there is an $N \in \mathbb{N}$ such that for all $j \geq N$, $\phi^{(j)}(z) \notin K_\varepsilon$; alternatively, $d(\phi^{(j)}(z), \partial D) < \varepsilon$ for $j \geq N$. Hence, we have that $\phi^{(j)}(z) \rightarrow \partial D$ as $j \rightarrow \infty$ for all $z \in D$. Since $K(z, z) \rightarrow \infty$ as $z \rightarrow \infty$ by assumption, it must be the case that

$$\lim_{j \rightarrow \infty} K[\phi^{(j)}(z), \phi^{(j)}(z)] = \infty.$$

Consequently, for any $z \in D$, and for infinitely many values of j , we have that

$$(2) \quad K\{\phi[\phi^{(j)}(z)], \phi[\phi^{(j)}(z)]\} > K[\phi^{(j)}(z), \phi^{(j)}(z)] > 0.$$

It follows from the assumption that ψ is bounded away from 0 toward the boundary of D that there must be a $\delta > 0$ such that whenever $d(\xi, \partial D) < \delta$, $|\psi(\xi)| > \nu/2$. In addition, for sufficiently large j , we have that $d(\phi^{(j)}(z), \partial D) < \delta$, so that for these values of j , $|\psi[\phi^{(j)}(z)]| > \nu/2$. Therefore, for any $z \in D$, there is an $N \in \mathbb{N}$ such that for infinitely many values of $j \geq N$, the following inequality holds for infinitely many j 's:

$$|\psi(\phi^{(j)}(z))|^2 K\{\phi[\phi^{(j)}(z)], \phi[\phi^{(j)}(z)]\} > \frac{\nu^2}{4} K[\phi^{(j)}(z), \phi^{(j)}(z)] > 0.$$

Thus we have in particular that for any $z \in D$, there are infinitely many values of j such that

$$|\psi[\phi^{(j)}(z)]|^2 K[\phi^{(j)}(z), \phi^{(j)}(z)]^{-1} K\{\phi[\phi^{(j)}(z)], \phi[\phi^{(j)}(z)]\} > \frac{\nu^2}{4}.$$

Denote this sequence of values of j by $(j_k)_{k \in \mathbb{N}}$. Then we have that $\phi^{(j_k)}(z) \rightarrow \partial D$ as $k \rightarrow \infty$. This fact, in combination with the fact that the above inequality for any $w \in D$ holds for the subsequence $(j_k)_{k \in \mathbb{N}}$ of \mathbb{N} , leads to a contradiction of Equation (1). Hence, the assumption that ϕ has no fixed points is false.

To show that ϕ has only one fixed point, assume to the contrary that ϕ has more than one fixed point. By a result of Vigué, the fixed point set of a holomorphic self-map of a bounded, convex domain in \mathbb{C}^n is a connected, analytic submanifold of that domain (see [Cl2, Thm. 4.1] or [Vi2]). Note that since the fixed point set of ϕ is not a singleton, then this set is, in particular, uncountable. Denote this set of fixed points by \mathcal{F} . We then have that

$$(3) \quad W_{\psi,\phi}^*(K_a) = \overline{\psi(a)}K_{\phi(a)} = \overline{\psi(a)}K_a \text{ for all } a \in \mathcal{F}.$$

Therefore, for all $a \in \mathcal{F}$, we have that $\overline{\psi(a)}$ is an eigenvalue of the compact operator $W_{\psi,\phi}^*$. Since ψ is continuous and \mathcal{F} is a connected analytic manifold in \mathbb{C}^n , it must be the case that $\psi(\mathcal{F})$ must be either a singleton or uncountable.

First, assume that $\psi(\mathcal{F})$ is a singleton $\{\lambda\}$, so that Condition (3) becomes

$$W_{\psi,\phi}^*(K_a) = \bar{\lambda}K_{\phi(a)} = \bar{\lambda}K_a \text{ for all } a \in \mathcal{F}.$$

Suppose that $\lambda = 0$. Then $\ker W_{\psi,\phi}^*$ has uncountable dimension, since $\{K_a : a \in D\}$ is linearly independent, thus contradicting the separability of \mathcal{Y} . Therefore, assume that $\lambda \neq 0$. Since $\{K_a : a \in D\}$ is a linearly independent set, it follows that the $\bar{\lambda}$ -eigenspace of $W_{\psi,\phi}^*$ has infinite dimension. However, by [Con, Thm. 7.1], this infiniteness contradicts the compactness of $W_{\psi,\phi}^*$ on \mathcal{Y}^* .

Next, assume that $\psi(\mathcal{F})$ is uncountable. Then by Condition (3), $W_{\psi,\phi}^*$ has uncountably many eigenvalues $\overline{\psi(a)}$ with $a \in \mathcal{F}$. Now since \mathcal{Y} contains the polynomials and is, therefore, infinite-dimensional, \mathcal{Y}^* is also infinite-dimensional. Therefore, the compact operator $W_{\psi,\phi}^*$ has a countably many eigenvalues [Con, p. 214], and we have again obtained a contradiction.

Hence, it must be the case that our assumption that ϕ has more than one fixed point is false. \square

4. SOME REMARKS AND RELATED, OPEN QUESTIONS

(1) Note that if $D = \Delta$ or \mathbb{B}_n , the fixed point of ϕ in Theorem 3.1 is precisely the so-called Denjoy-Wolff point of ϕ , to which the iterates of ϕ converge uniformly on compacta. It is natural to extend this uniform convergence to the general setting of Theorem 3.1.

(2) As stated in [Cl2], an interesting aspect of the above result is that in the case when $D = \Delta^n$, the Denjoy-Wolff theorem fails, and there is no unique “Denjoy-Wolff point”. Nevertheless, the above fixed point theorem holds even for reducible convex domains such as Δ^n .

(2) The convexity of D in the proof of Theorem 3.1 is used in two places: (a) to establish that if ϕ has no interior fixed points, the iterates of ϕ diverge compactly, and (b) to establish the assertion that the fixed point set of ϕ when D is convex is a *connected* analytic submanifold of D . It is therefore of interest to determine to what extent the hypothesis of convexity can be weakened in such a way that tasks (a) and (b) can still be simultaneously completed.

(3) Let G be a simply connected region that is properly contained in \mathbb{C} , and suppose that $\tau : \Delta \rightarrow \mathbb{C}$ is the Riemann mapping for G . Let $H^2(G)$ be the Hardy space of functions $f : G \rightarrow \mathbb{C}$ that are analytic and satisfy

$$\sup_{0 < r < 1} \int_{\tau(\{z \in \Delta : |z|=r\})} |f(z)|^2 |dz| < \infty.$$

In [ShSm] it is shown that if C_ϕ is compact on $H^2(G)$ for some analytic $\phi : \Delta \rightarrow \Delta$, then ϕ must have a unique fixed point in G . Of course, such a domain G can have boundary portions that are concave, though all domains in \mathbb{C} are trivially pseudoconvex [K2]. On the other hand, as is well known, the Riemann mapping theorem does not extend to several complex variables, and the proof in [ShSm] does seem to rely on the Denjoy-Wolff theory that is inherent from the convexity of Δ .

(4) Concerning the goal of obtaining compact divergence results for domains that are not necessarily convex, we list some of the best known results: First, in [H], it is proven by X. Huang that the iterates of a holomorphic self-map of a topologically contractible, strictly pseudoconvex domain form a compactly divergent sequence; however, in [Ab2], M. Abate showed the existence of a holomorphic self-map of a topologically contractible *pseudoconvex* domain such that the map's iterates do not compactly diverge.

(5) Note that in the proof of Theorem 3.1, all that was needed from Vigué's theorem is the assertion that if the fixed point set of a holomorphic self-map of a convex domain is non-empty, then it either contains one point or uncountably many points. Vigué in [Vi1] has shown that the fixed point set of a holomorphic self-map of any bounded domain D (note that "convex" is omitted!) in \mathbb{C}^n is also an analytic submanifold of D , but it is an interesting and open question as to whether or not the fixed point set in this case is necessarily connected for general bounded domains besides the convex ones.

M. Abate has conjectured that the answer is affirmative for a topologically contractible, strictly pseudoconvex domain. A resolution of this conjecture, together with Huang's previously mentioned result [H]

concerning compact divergence of iterates of holomorphic self-maps on topologically contractible pseudoconvex domains, would imply that Theorem 3.1 extends to these domains.

(6) It is natural to wonder if a holomorphic self-map of a domain in \mathbb{C}^n can have more than one but finitely many fixed points. This question is easy to answer if we allow the domain to be non-contractible. Let $r \in (0, 1)$, and let D be the n -fold topological product of the annulus with inner radius r and outer radius $1/r$. Define $\phi : D \rightarrow D$ be given by $\phi(z_1, z_2, \dots, z_n) = (z_1^{-1}, z_2^{-1}, \dots, z_n^{-1})$. ϕ has exactly 2^n fixed points, which are precisely the points whose entries are either 1 or -1 .

Thus, one can ask the following question: If D is a domain in \mathbb{C}^n and $k \in \mathbb{N}$, $k \neq 1$ is given, is there a holomorphic self-map of D with precisely k fixed points? In this direction, a result of J.-P. Vigué [K1, Thm. 1.7.6] states that an analytic self-map of a bounded domain in \mathbb{C} with three distinct fixed points is the identity mapping. The multivariable situation is quite different in this respect.

It would also be interesting to know how structured the fixed point set is in this case; for example, one can ask what are the possible Euclidean or fractal dimensions of such a fixed point set in pseudoconvex domains.

(7) For the weighted Hardy spaces $H_b^2(\Delta)$ of the unit disk in $\Delta \in \mathbb{C}$, the Hardy spaces $H^2(D)$, and weighted Bergman spaces $A_\alpha^2(D)$, where D is either \mathbb{B}_n , Δ^n , or more generally, any bounded symmetric domain in its Harish-Chandra realization (see [Cl2]), the reproducing kernel K satisfies $K(z, z) \rightarrow \infty$ as $z \rightarrow \Delta$ (respectively, $z \rightarrow D$), so the following fact, which extends the fixed point results in [CaSc] and [G], is an immediate consequence of Theorem 3.1:

Corollary 4.1. *Suppose that \mathcal{Y} is either the Hardy space $H^2(D)$ or the weighted Bergman space $A_\alpha^2(D)$ of a bounded symmetric domain D with $\alpha < \alpha_D$, where α_D is a certain critical value that depends on D (cf. [Cl2]), and assume that $\psi : D \rightarrow \mathbb{C}$ is analytic and bounded away from zero. Suppose that $\phi : D \rightarrow D$ is holomorphic, and let $W_{\psi, \phi}$ be compact on \mathcal{Y} . Then ϕ has a unique fixed-point in D . This result also holds when $D = \Delta$ and $\mathcal{Y} = H_b^2(\Delta)$.*

Proof. The assertions about $H^2(D)$ and $A_\alpha^2(D)$ immediately follow from Theorem 3.1 and the fact that their reproducing kernels approach infinity along the diagonal $\{(z, z) : z \in D\}$ as $z \rightarrow D$ (see [Cl2]). The assertion about $\mathcal{Y} = H_b^2(\Delta)$ also immediately follows from Theorem 3.1 and the fact that the assumed condition on the sequence $(b_j)_{j \in \mathbb{N}}$ implies

that the reproducing kernel K for $H_b^2(\Delta)$ satisfies the same singularity property toward the boundary along the diagonal (cf. [Cl2]). \square

5. ACKNOWLEDGMENT

The author would like to thank M. Abate for correspondences that led to some portions of Remarks (4)-(6) above. Thanks are also extended to W. Sheng and T. Oikhberg for helpful comments.

REFERENCES

- [Ab] M. Abate. *Iteration Theory of Holomorphic Maps on Taut Manifolds*, Mediterranean Press, Rende, 1989.
- [Ab2] M. Abate. *Holomorphic actions on contractible domains without fixed points*, Math. Z. **211** (1992), 547-555.
- [B] S. Banach. *Theorie des Operations Lineares*, Chelsea, Warsaw, 1932.
- [CaSc] J. Caughran/H. J. Schwartz. *Spectra of compact composition operators*, Proc. Amer. Math. Soc. **51**, (1975), 127-130.
- [ChMe] C.-H. Chu/P. Mellon. *Iteration of compact holomorphic maps on a Hilbert ball*, Proc. Amer. Math. Soc. **125** (1997), no. 6, 1771-1777.
- [CiW] J. A. Cima, W. R. Wogen. *On isometries of the Bloch space*, Illinois J. Math. **24** (2) (1980), 313-316.
- [Cl2] D. D. Clahane. *Spectra of compact composition operators over bounded symmetric domains*, Integral Equations Operator Theory **51** (2005), no. 1, 41-56.
- [Con] J. B. Conway. *A Course in Functional Analysis*, second edition, Springer-Verlag, New York, 1990.
- [CowMac] C. C. Cowen/B. D. MacCluer. *Composition Operators on Spaces of Analytic Functions*, CRC Press, Boca Raton, 1995.
- [dRW] K. DeLeeuw, W. Rudin, J. Wermer. *The isometries of some function spaces*, Proc. Amer. Math. Soc. **11** (1960), 485-694.
- [FlJ] R. J. Fleming/ J. E. Jamison. *Isometries on Banach spaces: function spaces*, Monographs and Surveys in Pure and Applied Mathematics, vol. 129, Chapman and Hall/CRC, London, Boca Raton, FL, 2002.
- [Fo] F. Forelli. *The isometries of H^p* , Canad. J. Math. **16** (1964), 721-728.
- [G] G. Gunatillake. *The spectrum of a compact weighted composition operator*, Proc. Amer. Math. Soc., to appear.
- [H] X. Huang. *A non-degeneracy property of extremal mappings and iterates of holomorphic self-mappings*, Ann. Scuola Norm. Pisa Cl. Sci. (4) **21** (1994), no. 3, 399-413.
- [K1] S. G. Krantz. *Geometric Analysis and Function Spaces*, CBMS Regional Conference Series in Mathematics, **81**, Amer. Math. Soc., Providence, 1993.
- [K2] S. G. Krantz. *Function Theory of Several Complex Variables*, Amer. Math. Soc., Providence, 2001.
- [Mac2] B. D. MacCluer. *Spectra of compact composition operators on $H^p(B_n)$* , Analysis **4** (1984), 87-103.
- [ShSm] J. H. Shapiro/W. Smith. *Hardy spaces that support no compact composition operators*, J. Funct. Anal. **205** (2003), no. 1, 62-89.
- [S] M. Stone. *Application of the theory of Boolean rings in topology*, Trans. Amer. Math. Soc. **41** (1937) 375-481.

- [Vi1] J.-P. Vigué. *Sur les points fixes d'applications holomorphes. (French) [On the fixed points of holomorphic mappings]*, C. R. Acad. Sci. Paris Sr. I Math. **303**, (1986), no. 18, 927-930.
- [Vi2] J.-P. Vigué. *Points fixes d'applications holomorphes dans un domaine borné convexe de C^n* [Fixed points of holomorphic mappings in a bounded convex domain in \mathbf{C}^n], Trans. Amer. Math. Soc. **289** (1985), no. 1, 345-353.

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CALIFORNIA
RIVERSIDE, CA 92521.

E-mail address: `dclahane@math.ucr.edu`